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Reflection and Rotation Groups in Ordinary Space

The symmetry of a regular polyhedron or Archimedean solid $\mathcal{P}$ in $\mathbb{R}^3$ is embodied in its symmetry group $\Gamma = \Gamma(\mathcal{P})$. I will discuss and enumerate the relevant groups Γ, which turn out to be generated by reflections, or occasionally just rotations, in a very special way. (See [1] for some indication of the very rich mathematics that this leads to.)

Next we will see how to start with the group Γ and from it reconstruct the polyhedron $\mathcal{P}$, using a beautiful technique developed by Coxeter [2].

In many cases, this construction can be realized physically (well, optically) in a *kaleidoscope*. (I will attempt to bring an unbroken glass model from Canada.) Finally, I will show you a Maple program devised by Ryan Oulton, an undergraduate at UNB, which gives you assembly instructions based on a very simple input. For example, the command

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kaleidoscope(3,5,1,1,0)
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produces a motif for building a *truncated icosahedron* (topologically, a *soccer ball*). But it is fun to avoid computers, if you wish.

References

1. H.S.M. Coxeter, *Regular Complex Polytopes* (2nd Edition), CUP, Cambridge, 1991; see pages 9–25 in particular.

2. H.S.M. Coxeter, *Wythoff's construction for uniform polytopes*, Proc. London Math. Soc. **38** (1935), 327–339. [Reprinted in *The Beauty of Geometry: Twelve Essays*, Dover, NY, 1999.]

3. B. Monson: trawling through my web-site you can find relevant materials at
`http://www.math.unb.ca/~barry/summer2011/indexsum.html`
or at
`http://www.math.unb.ca/~barry/fields/`