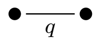
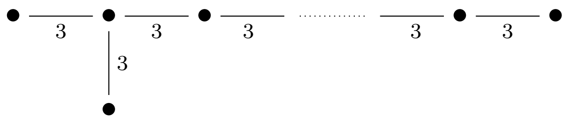
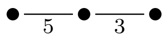
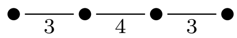
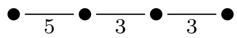
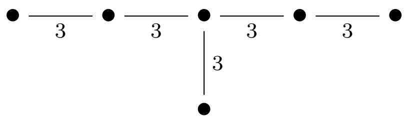
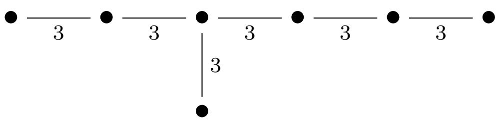
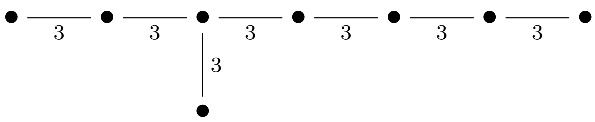
$\Delta_c(G)$	G	$\|G\|$
	$I_2(q)$, $q \geqslant 2$ (dihedral)	$2q$
 ($n \geqslant 1$ nodes)	$A_n \simeq S_{n+1}$ (symmetric gp)	$(n+1)!$
 ($n \geqslant 2$ nodes)	$B_n \simeq B_n(2)$	$2^n \cdot n!$
 ($n \geqslant 2$ nodes)	$D_n \simeq D_n(2)$	$2^{n-1} \cdot n!$
	$H_3 \simeq \mathrm{Alt}_5 \times C_2$ (icosahedral)	120
	F_4	1152
	H_4	$14400 = 120^2$
	E_6	$51840 = 72 \cdot 6!$
	E_7	$2903040 = 8 \cdot 72 \cdot 7!$
	E_8	$696729600 = 240 \cdot 72 \cdot 8!$

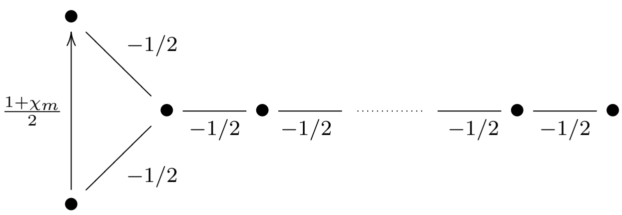
Coxeter Diagrams $\Delta_c(G)$ for Irreducible Finite Coxeter Groups G.

Remarks.

1. $A_1 \simeq C_2$; $A_2 \simeq I_2(3)$; $A_3 \simeq D_3$ is the tetrahedral reflection group $[3, 3]$.
2. $B_2 \simeq I_2(4)$; B_3 is the octahedral reflection group $[4, 3]$.
3. D_n has index 2 in B_n; $D_2 \simeq C_2 \times C_2$.
4. H_3 is the icosahedral reflection group $[5, 3]$; we might say $H_2 = I_2(5)$.

Here are the finite, irreducible *unitary* groups generated by reflections of period 2 (not already on the above list of real forms). The diagrams incorporate different information (presentations not simply of Coxeter type!):

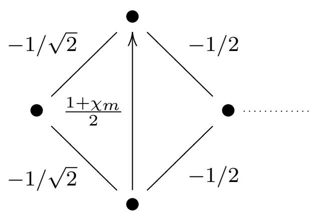
Imprimitive Cases The group $D_n(m) = G(m, m, n)$, where $m \geqslant 2$, has order $m^{n-1}n!$ and generalizes the Coxeter group of type D_n, which is actually isomorphic to $D_n(2)$. The corresponding *root diagram*



(1)

is a variant of the Coxeter diagram and encodes the details of the essentially unique Hermitian form left invariant by $G = G(m, m, n)$ Here χ_m is a primitive mth root of unity.

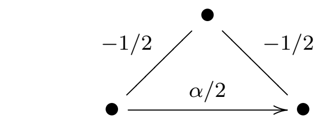
The other type of imprimitive group is $B_n(m) = G(m, \frac{m}{2}, n)$, for *even* integers $m \geqslant 2$. This generalization of the Coxeter group B_n has order $2m^{n-1}n!$. To generate $B_n(m)$ we usually require $n + 1$ reflections, say $r_0, \ldots, r_{n-1}$ corresponding to the n nodes in (1), together with another reflection r_-, whose node is attached in (1) as follows:

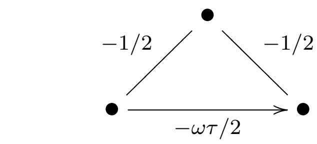


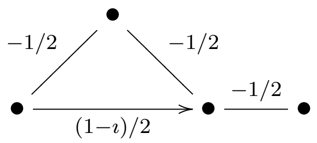
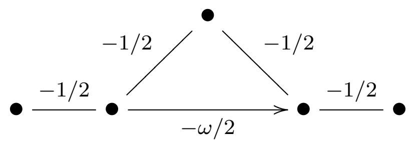
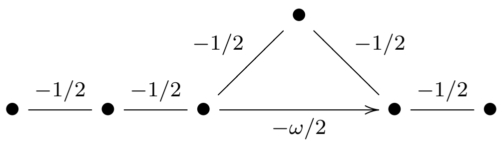
(2)

However, when $m = 2$, the generator r_0 becomes superfluous and we do get $B_n \simeq B_n(2)$.

Finally we look at the *primitive* groups $J_3(4)$, $J_3(5)$, N_4, EN_4, K_5 and K_6, whose root diagrams appear in In the notation of we have $J_3(4) = [1\,1\,1^4]^4$, $J_3(5) = [1\,1\,1^5]^4$, $N_4 = [1\,1\,2]^4$, $K_5 = [2\,1\,2]^3$ and $K_6 = [2\,1\,3]^3$; these groups require n generating reflections, whereas EN_4 requires $n + 1 (= 5)$.

<table>
<tr><th>$\Delta_c(G)$</th><th>G</th><th>$|G|$</th></tr>
<tr><td>

where $\alpha^2 - \alpha + 2 = 0$</td><td>$J_3(4) \simeq [111^4]^4$</td><td>336</td></tr>
<tr><td>

where $\omega = (-1 + \imath\sqrt{3})/2$, $\tau = (1 + \sqrt{5})/2$</td><td>$J_3(5) \simeq [111^4]^5$</td><td>2160</td></tr>
<tr><td>
</td><td>$N_4 \simeq [112]^4$</td><td>$64 \cdot 5!$</td></tr>
<tr><td>
</td><td>$K_5 \simeq [212]^3$</td><td>$72 \cdot 6!$</td></tr>
<tr><td>
</td><td>$K_6 \simeq [213]^3$</td><td>$108 \cdot 9!$</td></tr>
</table>

Root Diagrams for the Remaining Irreducible, Primitive Unitary Groups Generated by Reflections of Period 2.

Our set-up: $x \cdot y$ is a Hermitian form on complex n-space V. Thus $x \cdot y = \overline{y \cdot x}$; $x \cdot (ty) = t(x \cdot y)$; $(tx) \cdot y = \bar{t}(x \cdot y)$, for $x, y \in V$, $t \in \mathbb{C}$, etc. The (involutory) unitary reflection r with root $a \in V \setminus \{o\}$ is

$$r(x) = x - \frac{2(a \cdot x)}{a \cdot a}\, a \; .$$

(For period m replace the '2' by $1 - \chi$, where χ is a primitive mth root of unity.)

In a root diagram above, the nodes correspond to a normalized basis $a_0, \ldots, a_{n-1}$ of unitary n-space V. The Hermitian form is defined by its Gram matrix for this basis. Thus all $a_j \cdot a_j = 1$ and $a_i \cdot a_j = 0$ if nodes i, j are non-adjacent. Finally, when there is an arrow from node i to node j with branch label $t \neq 0$, then $a_i \cdot a_j = t$ (so $a_j \cdot a_i = \bar{t}$). Branches with real labels need not be directed.

The corresponding unitary reflection group $G = \langle r_0, \ldots, r_{n-1} \rangle$, where r_j has root a_j. Groups EN_4 and $B_n(m)$, for $m = 2k \geqslant 4$, require one extra generator.