Why Support Research in Mathematics?

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As a mathematician, I am frequently asked to explain what mathematical research is. Despite the widespread applications of technology in our everyday lives, nobody ever sees an equation when checking the weather channel or turning on a cell phone. Even people who have studied mathematics in university while pursuing a degree in science or engineering may still look upon mathematics as a collection of computational techniques. These are, of course, an integral part of mathematics. But they represent the tip of the iceberg, while the ballast is provided by a huge body of understanding that remains hidden beneath the surface from non-practitioners. The reality is that mathematics provides the framework for understanding almost any complicated phenomenon. Some mathematicians work directly on these interactions with other fields, but many others pursue problems without any direct application in mind. It is often the latter pursuits which paradoxically may have the greatest impact in the long run.

Mathematics is everywhere.
It is difficult to identify any item produced by modern industry where mathematics did not play a crucial role somewhere along the line. In manufacturing, advanced mathematical techniques are used to model and test products on computers, to visualize design, to control robots, and to optimize production techniques. The computer itself was conceived (and early working models built) by mathematicians. Interpretations of hidden structure are made possible by the measurement of reflected waves—from MRIs to earthquakes—to locate brain tumours, to find oil, and to detect planets around distant suns. All of this was developed from mathematics which had its roots three hundred years ago. Along the way, the study moved from a simple physical situation deep into mathematical abstraction and back out to physics again.

Most computer and cell phone equipment uses mathematical techniques for encryption, not just for security but also for data compression and for ensured accuracy of transmission. The world telephone network requires extremely sophisticated algorithms to route with speed and efficiency millions of signals simultaneously, without the users' awareness.

Statistics is an indispensable tool in business, science, and even public policy. Indeed, we are inundated with statistics daily in the news. How do we actually know that 46.7 per cent of Canadians support the federal Liberal party?—did they ask you? Biologists are trying to understand the human genome with thirty thousand genes and potentially nine billion pairs of protein interactions as a result. They produce huge quantities of data that are meaningless until analysed by statistical methods. The banking and insurance industries rely on mathematics and statistics to evaluate risk, to make actuarial assessments of mortality, and to maintain proper cash reserves.

Mathematics is a way of thinking.
Mathematics has been called "the language of science." This is not just a matter of vocabulary. Most complicated physical phenomena follow rather simple principles, generally expressed in precise mathematical formulas. What mathematics does for science is not just shorthand-—it is a very different way of looking at and thinking about the problem. Indeed the mathematization of a "real" problem generally involves abstracting out general structures. Any pesky details such as actual measurements may be replaced by arbitrary constants. In this way, whole families of related problems are considered simultaneously, with the original problem merely a single instance.

At this point, the mathematics can take on a life of its own. Mathematicians may attack the problem by making simplifying assumptions that alter the original problem. In solving this simpler problem, they may identify important properties that can be measured and which yield key information about the whole system. These ideas then feed back into the larger context. Predictions from both the behaviour in the simpler theoretical models and from computational experiments in the more complicated situations lead mathematicians to make conjectures about the general case. Some such conjectures are so incisive and their solutions have such profound implications that they become world-famous problems. And eventually someone solves them! Frequently, many years after the original problem made its way from the real world to the symbolic world of mathematics, the answer comes back and causes a revolution in thinking.

Mathematics is predictive.
Most people understand mathematics as a tool for calculation. Its public side is a huge "bag of tricks" for determining something you need to know from certain measurements. For example, how much fuel is needed for a 747 airplane with two hundred passengers and twenty tons of cargo to fly from New York to London? But even more important is the fact that mathematical models predict that certain things must happen, not just quantitatively but also qualitatively. Such predictions can often indicate new phenomena that scientists can see once mathematics tells them where to look for it.

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Equally important but more subtle, mathematics can often tell you that something is impossible. When most people say that something is impossible, they mean that, in their experience, it cannot be done and, further, they cannot imagine how it might be accomplished. (Most modern technology was in that category not so long ago.) You are always left with the nagging feeling that perhaps you are just not smart enough. But a mathematician can often say with absolute certainty that something cannot be done, not just with today's resources, but even with unimagined advances. Better yet, one can often identify an obstruction—a quantifiable measurement that calculates when one can do something and when one cannot. For example, a torus (doughnut) cannot be smoothly deformed into a sphere.

Finally, in a perverse turn of the screw, sometimes in attempting to show that a certain property is an obstruction, you find out that it is not! Overcoming this obstacle in its simplest instance can often lead to a solution in great generality.

Mathematics is beautiful.
Mathematics is an art as well as a science. Hidden in rather simple rules (axioms) for a class of mathematical objects are many amazing consequences. Deep connections between ideas from different areas often result in quite a simple picture of how things fit together. The complicated morass of isolated examples becomes understandable in the big picture through the lens of a new idea.

Most mathematicians have a strong aesthetic sense of what good mathematics is. We distinguish a clean elegant argument from a grungy brute force approach, and we distinguish deep insights from routine extensions of old ideas. Some difficult new ideas arrive fully polished, but usually it is a messier business. The first breakthrough is a rough diamond. Full understanding often comes later, as others analyse the new technique, simplify, generalize, and polish it. Certain aspects hidden in a clever but opaque argument may be exposed and clarified. Over time, some results prove to be of central importance while others fall by the wayside, correct but of little lasting interest. Eventually the synthesis ends up in a textbook and becomes part of the standard lore of practitioners as well as experts.

Mathematics is hard to explain.
I heard a talk by Jay Ingram, moderator of the television science series @discovery.ca. He described how difficult it is to explain science to the layman because the context is not there. Most people do not know the vocabulary of science and are unable to recognize scientific phenomena in their everyday lives. Well, in mathematics, we try to get our context by making a connection to science!

People see advances in technology every day. Engineering and drug companies get most of the credit. Behind the scenes, you may envisage an engineer doing wind tunnel experiments or a biologist identifying the gene that controls breast cancer. But it is not easy to see how a mathematician who can classify "finite simple groups" fits into the picture. Indeed, I would find it difficult just to explain what a "simple group" is to a non-mathematician, yet the classification of such groups is a major achievement of twentieth-century mathematics.

Why support mathematical research?
Since technology is advancing rapidly, turning ideas from science into useful products, you might believe that the best use of resources is to pour funds into that transition. In fact, this last step is both the best funded and least scientific. Many entrepreneurs, from basement inventors to multinational companies, put a lot of effort into producing technology for profit. But while real problems are overcome at this level, they are usually of an incremental nature.

Fundamental advances in science often occur unexpectedly. Of course, scientists expect to make advances, and are generally driven by long-term goals of understanding. Every once in a while, however, scientists are in exactly the right place at the right time. Then their experience and knowledge enable them to recognize and develop the fortuitous insight. Trying to predict in advance which investigations will produce important applications is not only futile but also dangerous. Both scientists and policy makers have a consistently bad record of charting the important areas of future development. Funding the most promising application instead of the most promising scientist chooses good public relations over good science.

Mathematics is often remote from the big payoff of a saleable product. Increasingly, however, sophisticated ideas from mathematics—whether new or old—play a critical role in all of science and engineering. The theoretical advances of today feed the practical advances of tomorrow.

Moreover, mathematics is relatively cheap!
There are no big labs, no cyclotrons, or gene sequencers, or x-ray spectrographs. Mathematicians travel with their ideas in their heads, with pencils, paper, and of course, computers. Contrary to popular mythology, mathematicians are very sociable; they love to talk about their mathematics (to other mathematicians, of course). If you bring together a group of talented mathematicians interested in a common problem, a lot will happen. A number of international mathematics institutes such the Fields Institute in Toronto have been established specifically to facilitate this kind of interaction. We seek out opportunities in any area of mathematical science where there is promising activity.

Mathematics is a living breathing growing subject. It is the fuel for science and technology. At work for the most part behind the scenes, mathematics makes it possible to understand the world we live in.

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