Part of Rubik's Cube

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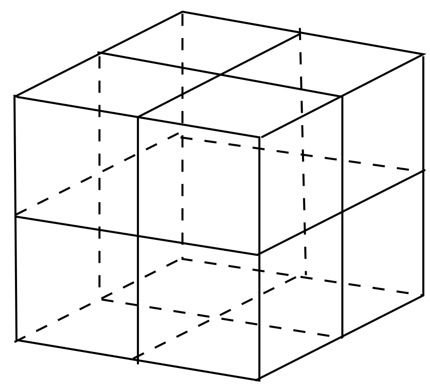
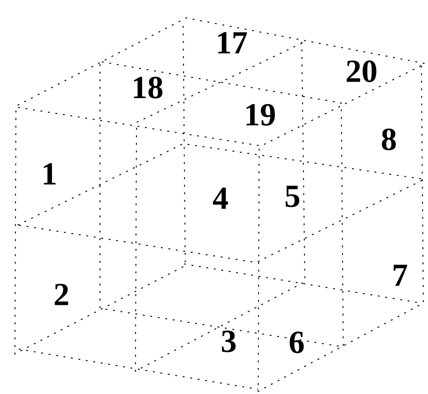
1. You have all seen *Rubik's Cube*. This astonishing mechanical device has a rich group structure.

   To begin with, we will look at the simpler puzzle obtained by extracting the 8 corner cubits from Rubik's cube and reassembling them into a $2 \times 2 \times 2$ cube. Basically this is the same as focussing our attention on the corner cubits only in Rubik's original. What we see then is something like this:

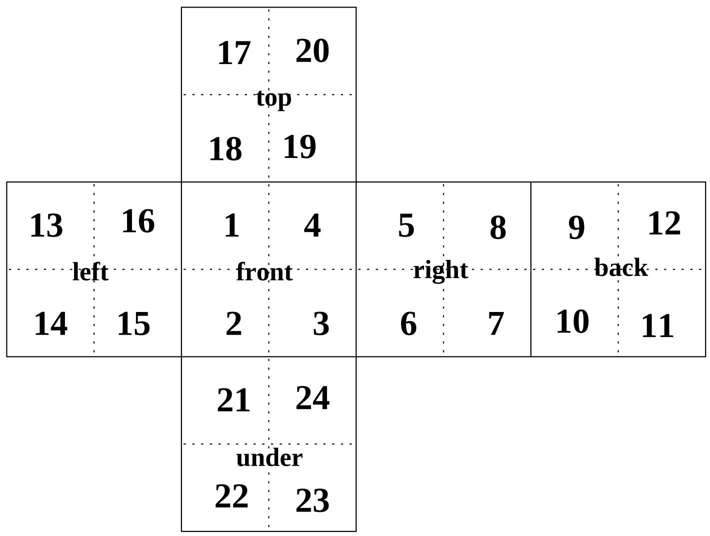
   The reason the resulting mechanical group G is so big is that we are actually permuting $24 = 6 \cdot 4$ things, namely the 1×1 squares on the square faces.

   To describe the group we again need fixed background labels, now in space. Think of the cube as lying snugly inside a slightly bigger and immovable package, whose subsquares we have labelled:

We then describe the action in the mechanical cube with respect to these fixed labels in space.

2. To establish these labels more clearly, let's unfold the cube, put in what labels we have above, followed by the remaining labels:



3. It is clear that any manipulation of the cube involves repeated use of the individual turns about the square faces. Consecutive manipulations correspond to multiplying the permutations of the faces.

   An anticlockwise quarter-turn can be repeated twice to give a half-turn or three times to give the inverse clockwise turn. Thus to generate the group we

   need only the six quarter turns for the faces,
   taken anticlockwise as we look *from outside into the cube.*

   Now it is just a matter of patience to represent each turn as a permutation of subsquares. Notice that each quarter turn fixes half of these subsquares and moves the other 12 in three cycles of four. In the end we get

left	$L = (13, 14, 15, 16)(1, 17, 11, 21)(2, 18, 12, 22)$
right	$R = (5, 6, 7, 8)(19, 3, 23, 9)(20, 4, 24, 10)$
top	$T = (17, 18, 19, 20)(1, 5, 9, 13)(4, 8, 12, 16)$
under	$U = (21, 22, 23, 24)(2, 14, 10, 6)(3, 15, 11, 7)$
front	$F = (1, 2, 3, 4)(18, 15, 24, 5)(19, 16, 21, 6)$
back	$B = (9, 10, 11, 12)(20, 7, 22, 13)(17, 8, 23, 14)$