Rubik's (Original) Cube
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1. In Rubik's actual cube, the 8 corner cubits behave just like those in the $2 \times 2 \times 2$ cube, which we have looked at separately. But in addition we have the 12 edge cubits to contend with. To some extent (but not completely!) these 12 operate independently of the original 8.

   The centre squares on each of the six faces of the cube do turn. But we can essentially take them to be fixed and hence ignore them. Basically this means one keeps the face colours fixed in spatial orientation.

2. The full mechanical group M is now much larger than for the $2 \times 2 \times 2$ cube. Its order is

$$
\begin{aligned}
43252003274489856000 &= 2^{27} \cdot 3^{14} \cdot 5^3 \cdot 7^2 \cdot 11 \\
&= 2^{20} \cdot 3^5 \cdot 5^2 \cdot 7 \cdot 11 \cdot 88179840 \ .
\end{aligned}
$$

   You see 88179840 as the order of the 'corner subgroup' G from before.

3. To describe the group M we still need fixed background labels, only now there are $48 = 24 + 24$ of them. I'll take the second set of 24 (for the edge cubit squares) to be $30 \ldots, 53$. (see over).

			17	46	20						
			47	top	49						
			18	48	19						
13	42	16	1	30	4	5	34	8	9	38	12
43	left	45	31	front	33	35	right	37	39	back	41
14	44	15	2	32	3	6	36	7	10	40	11
			21	50	24						
			51	under	53						
			22	52	23						

4. The six quarter turns for the faces, still taken anticlockwise as we look *from outside the cube onto a square face* are

$$
\begin{array}{rl}
\text{left} & L = (1, 17, 11, 21)(2, 18, 12, 22)(13, 14, 15, 16)(31, 47, 41, 51)(42, 43, 44, 45) \\
\text{right} & R = (3, 23, 9, 19)(4, 24, 10, 20)(5, 6, 7, 8)(33, 53, 39, 49)(34, 35, 36, 37) \\
\text{top} & T = (1, 5, 9, 13)(4, 8, 12, 16)(17, 18, 19, 20)(30, 34, 38, 42)(46, 47, 48, 49) \\
\text{under} & U = (2, 14, 10, 6)(3, 15, 11, 7)(21, 22, 23, 24)(32, 44, 40, 36)(50, 51, 52, 53) \\
\text{front} & F = (1, 2, 3, 4)(5, 18, 15, 24)(6, 19, 16, 21)(30, 31, 32, 33)(35, 48, 45, 50) \\
\text{back} & B = (7, 22, 13, 20)(8, 23, 14, 17)(9, 10, 11, 12)(37, 52, 43, 46)(38, 39, 40, 41)
\end{array}
$$